Nonparametric Methods: Nominal Level Hypothesis Tests

Chapter 15

Learning Objectives

LO15-1 Test a hypothesis about a population proportion

LO15-2 Test a hypothesis about two population proportions

LO15-3 Test a hypothesis comparing an observed set of frequencies to an expected frequency distribution

LO15-4 Explain the limitations of using the chi-square statistic in goodness-of-fit tests

LO15-5 Test a hypothesis that an observed frequency distribution is normally distributed

LO15-6 Perform a chi-square test for independence on a contingency table

Test a Hypothesis of a Population Proportion

- Recall that a proportion is the ratio of the number of successes to the number of observations
- Examples
- Historically, GM reports that 70% of leased vehicles are returned with less than 36,000 miles; a recent sample of 200 found that 158 had less than 36,000 miles. Has the proportion increased?
- Able Moving and Storage advises its clients that their household goods will be delivered in 3 to 5 days for a long-distance move. Records show this is true 90% of the time. A recent sample of 200 moves found that they were successful 190 times. Has the success rate increased?

Hypothesis Test of a Population Proportion

- To test a hypothesis about a population proportion, the binomial conditions must be met
  - The sample data collected are the result of counts
  - The outcome of the experiment must be classified in one of two mutually exclusive classes
  - The probability of a success is the same for each trial
  - The trials are independent
  - Both $n\pi$ and $n(1 - \pi)$ must be at least 5
- The test statistic is

TEST OF HYPOTHESIS, ONE PROPORTION

$$z = \frac{p - \pi}{\sqrt{\frac{\pi(1 - \pi)}{n}}} \qquad (15\text{–}1)$$

Population Proportion Test Example

A Republican governor of a western state is thinking about running for reelection. Historically, to be reelected, a Republican needed at least 80% of the vote in the northern part of the state. The governor hires a polling organization to survey the voters there. The polling organization will poll 2,000 voters. Use a statistical hypothesis-testing procedure to assess the governor's chances of winning reelection.

Step 1: State the null and alternate hypothesis

$H_0: \pi \geq .80$

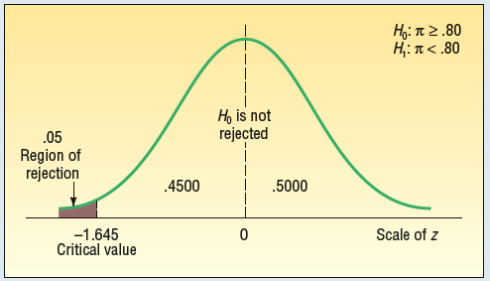
$H_1: \pi < .80$

Step 2: Select the level of significance, we'll use .05

Step 3: Select the test statistic, we use z

Step 4: Formulate the decision rule, reject H_0 if $z < -1.645$

Step 5: Take sample, make a decision, reject the null hypothesis



$$z = \frac{p - \pi}{\sqrt{\frac{\pi(1 - \pi)}{n}}} = \frac{\frac{1{,}550}{2{,}000} - .80}{\sqrt{\frac{.80(1 - .80)}{2{,}000}}} = \frac{.775 - .80}{\sqrt{.00008}} = -2.80$$

Step 6: Interpret, the governor does not have the votes to win

Two-Sample Tests about Proportions

- We can also test whether two samples came from populations with an equal proportion of successes
- Examples
- The vice president of human resources wishes to know whether there is a difference in the proportion of hourly employees who miss more than 5 days of work per year at the Atlanta and the Houston plants
- A consultant to the airline industry is investigating the fear of flying among adults. The company wishes to know if there is a difference between the proportion of men versus women who are fearful of flying

The Two-Sample Test of Proportions

- To test whether two samples came from populations with an equal proportion of successes
- First pool the two sample proportions using the following formula

POOLED PROPORTION

$$p_c = \frac{x_1 + x_2}{n_1 + n_2} \qquad [15\text{–}3]$$

- Then we compute the value of the test statistic from the following formula

TWO-SAMPLE TEST OF PROPORTIONS

$$z = \frac{p_1 - p_2}{\sqrt{\frac{p_c(1 - p_c)}{n_1} + \frac{p_c(1 - p_c)}{n_2}}} \qquad [15\text{–}2]$$

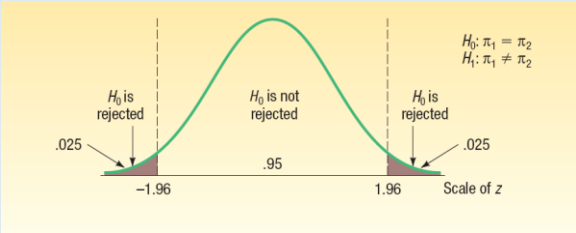
Two-Sample Tests about Proportions Example

Manelli Perfume Company recently developed a new fragrance that it plans to market under the name Heavenly. Market studies indicate that Heavenly has very good market potential. The sales department at Manelli is interested in whether there is a difference in the proportion of working women and stay-at-home women who would purchase Heavenly.

Step 1: State the null and the alternate hypothesis

$H_0: \pi_1 = \pi_2$

$H_1: \pi_1 \neq \pi_2$



Step 2: Select the level of significance, we select .05

Step 3: Determine the test statistic, we'll use z

Step 4: Formulate the decision rule, Reject H_0 if z < -1.96 or z > 1.96

Two-Sample Tests about Proportions Example Continued

Manelli Perfume Company samples 100 working women and 200 stay-at-home women to find out if the population proportions are equal. Each of the sampled women will be asked to smell Heavenly and indicate whether she likes it well enough to purchase a bottle.

Step 5: Take sample, make decision, reject H_0

$$p_1 = \frac{x_1}{n_1} = \frac{19}{100} = .19 \qquad p_2 = \frac{x_2}{n_2} = \frac{62}{200} = .31$$

$$p_c = \frac{x_1 + x_2}{n_1 + n_2} = \frac{19 + 62}{100 + 200} = \frac{81}{300} = 0.27$$

$$z = \frac{p_1 - p_2}{\sqrt{\frac{p_c(1 - p_c)}{n_1} + \frac{p_c(1 - p_c)}{n_2}}} = \frac{.19 - .31}{\sqrt{\frac{.27(1 - .27)}{100} + \frac{.27(1 - .27)}{200}}} = -2.207$$

Step 6: Interpret, working women and stay-at-home women will purchase Heavenly at different rates or proportions.

Goodness-of-Fit Test

- We can compare an observed frequency distribution to an expected frequency distribution
- Example
- An insurance company wishes to compare the historical distribution of policy types with a sample of 2,000 current policies
- Does the current distribution of policies "fit" this historical distribution, or has it changed

Policy Type	Percent
Whole life	40
Level term	25
Decreasing term	15
Other	20

Goodness-of-Fit Test Example

Bubba's Fish and Pasta is a chain of restaurants along the Gulf Coast of Florida. Bubba is considering adding steak to the menu. Before doing so, he hires a research firm to conduct a survey to find out what the patron's favorite meal is when eating out. Here are the results of the survey of 120 adults.

Favorite Entrée	Frequency
Chicken	32
Fish	24
Meat	35
Pasta	29
Total	120

Is it reasonable to conclude there is no preference among the four entrées?

Favorite Meal	Observed Frequency, f_o	Expected Frequency, f_o
Chicken	32	30
Fish	24	30
Meat	35	30
Pasta	29	30
Total	120	120

Is the difference in the number of times each entrée is selected due to chance, or should we conclude that the entrées are not equally preferred?

Goodness-of-Fit Test Example Continued

Step 1: State the null and the alternate hypothesis

H_0: There is no difference in the proportion of adults selecting each entrée

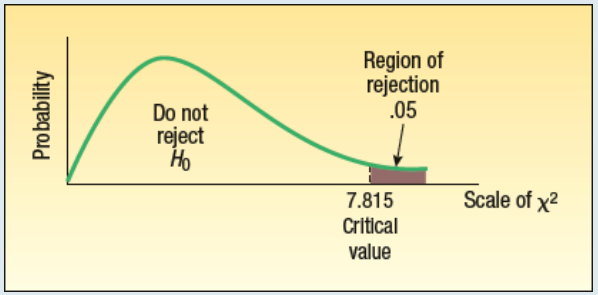
H_1: There is a difference in the proportion of adults selecting each entrée

Step 2: Select the level of significance, we select .05

Step 3: Select the test statistic, we'll use chi-square, χ^2

Step 4: Formulate the decision rule, reject H_0 if $\chi^2 > 7.815$

Degrees of Freedom *df*	Right-Tail Area			
	.10	.05	.02	.01
1	2.706	3.841	5.412	6.635
2	4.605	5.991	7.824	9.210
3	6.251	7.815	9.837	11.345
4	7.779	9.488	11.668	13.277
5	9.236	11.070	13.388	15.086



Goodness-of-Fit Test Example Concluded

Step 5: Take sample, make decision, do not reject H_0, 2.200 is not greater than 7.815

CHI-SQUARE TEST STATISTIC

$$\chi^2 = \Sigma\left[\frac{(f_o - f_e)^2}{f_e}\right] \qquad (15\text{–}4)$$

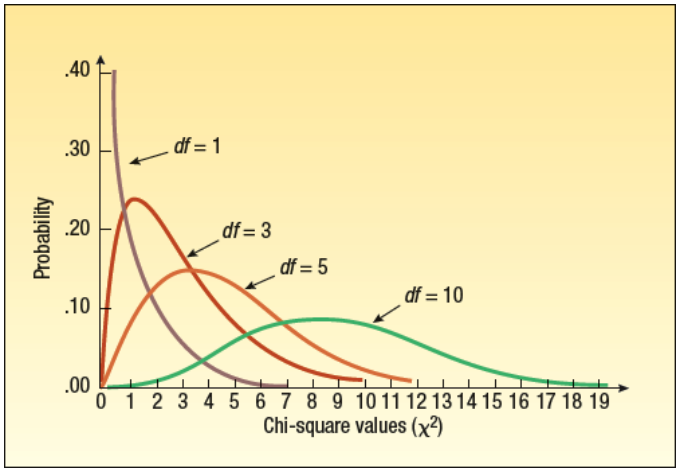
	A	B	C	D	E	F	G
1	Favorite Entrée	f_o	f_e	$(f_o - f_e)$	$(f_o - f_e)^2$	$(f_o - f_e)^2/f_e$	
2	Chicken	32	30	2	4	0.133	
3	Fish	24	30	-6	36	1.200	
4	Meat	35	30	5	25	0.833	
5	Pasta	29	30	-1	1	0.033	
6	Total	120	120			2.200	

χ^2 Value

Step 6: Interpret, the data do not suggest the preferences among the four entrées are different.

Chi-Square Characteristics

- The characteristics of the chi-square distribution are
  - The value of chi-square is never negative
  - There is a family of chi-square distributions
  - The chi-square distribution is positively skewed
  - As the degrees of freedom increase, the distribution approaches a normal distribution



Hypothesis Test of Unequal Expected Frequencies Example

The American Hospital Administration Association reports the number of times senior citizens are admitted to a hospital during a one-year period; 40% are not admitted, 30% are admitted once, 20% are admitted twice, and 10% are admitted 3 or more times.

Then, a survey of 150 residents of Bartow Estates, a community devoted to active seniors located in central Florida revealed 55 residents were not admitted, 50 were admitted once, 32 were admitted twice, and the rest in the survey were admitted three or more times. Can we conclude the survey at Bartow Estates is consistent with the information reported by the AHAA?

Number of Times Admitted	AHAA Percent of Total	Number of Bartow Residents (f_o)	Expected Number of Bartow Residents (f_e)
0	40	55	60
1	30	50	45
2	20	32	30
3 or more	10	13	15
Total	100	150	

Hypothesis Test of Unequal Expected Frequencies Example Continued

Step 1: State the null and alternate hypothesis

H_0: There is no difference between local and national experience for hospital admissions

H_1: There is a difference between local and national experience for hospital admissions

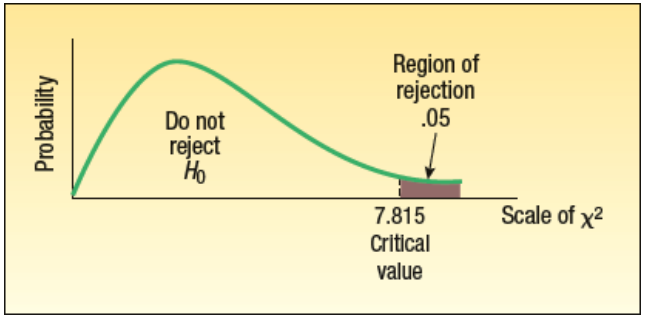
Step 2: Select the level of significance, we select .05

Step 3: Select the test statistic, we'll use χ^2

Step 4: Formulate the decision rule, reject H_0 if $\chi^2 > 7.815$

Step 5: Calculate the test statistic, make decision, do not reject H_0, $1.3723 < 7.815$

Step 6: Interpret, there is no evidence of a difference between the local and the national experience for hospital admissions



Number of Times Admitted	f_o	f_e	$(f_o - f_e)$	$(f_o - f_e)^2$	$(f_o - f_e)^2/f_e$
0	55	60	−5	25	0.4167
1	50	45	5	25	0.5556
2	32	30	2	4	0.1333
3 or more	13	15	−2	4	0.2667
Total	150				1.3723

χ^2 value

Limitations of Chi-Square

- If there is an unusually small frequency in a cell, chi-square might result in an erroneous conclusion
  - A very small number in the denominator, can make the quotient quite large
- For only two cells, the f_e should be at least 5
- For more than two cells, chi-square should not be used if more than 20% of the f_e cells have an expected frequency that is less than 5

Level of Management	f_o	f_e
Foreman	30	32
Supervisor	110	113
Manager	86	87
Middle management	23	24
Assistant vice president	5	2
Vice president	5	4
Senior vice president	4	1
Total	263	263

→

The issue can be resolved by combining categories if it is logical to do so. In this example, we combine the three vice president categories, which satisfies the 20% policy.

→

Level of Management	f_o	f_e
Foreman	30	32
Supervisor	110	113
Manager	86	87
Middle management	23	24
Vice president	14	7
Total	263	263

Goodness-of-Fit Test

- A goodness-of-fit test can be used to determine whether a sample of observations is from a normal population
  1. Calculate the mean and standard deviation of the sample data
  2. Group the data into a frequency distribution
  3. Convert the class limits to z values and find the standard normal probability distribution for each class
  4. For each class, find the expected normally distributed frequency by multiplying the standard normal probability distribution by the class frequency
  5. Calculate the chi-square goodness-of-fit statistic based on the observed and expected class frequencies
  6. Find the expected frequency in each cell by determining the product of the probability of finding a value in each cell by the total number of observations
  7. If we use the information on the sample mean and the sample standard deviation from the sample data, the degrees of freedom are k - 3

Hypothesis Test that a Distribution is Normal Example

We investigate whether the profit data of Applewood Auto Group follows the normal distribution. In chapter 3, we found the mean profit was $1,843.17 and the standard deviation was $643.63.

Profit	Frequency
$ 200 up to $ 600	8
600 up to 1,000	11
1,000 up to 1,400	23
1,400 up to 1,800	38
1,800 up to 2,200	45
2,200 up to 2,600	32
2,600 up to 3,000	19
3,000 up to 3,400	4
Total	180

Now, calculate z values to calculate the area of probability for each of the eight classes. This multiplied by the total, 180, will represent the expected frequencies for each class.

$$z = \frac{x - \bar{x}}{s} = \frac{\$200 - \$1,843.17}{\$643.63} = 2.55 \qquad z = \frac{x - \bar{x}}{s} = \frac{\$600 - \$1,843.17}{\$643.63} = 1.93$$

P(x < $200) = P(z < -2.55) = .5000 - .4946 = .0054

P($200 < x < $600) = P(-2.55 < z < -1.93) = .0268 - .0054 = .0214

Hypothesis Test that a Distribution is Normal Example Continued

Now, combine the classes that have $f_e < 5$.

Profit	z Values	Area	Found by	Expected Frequency
Under $200	Under −2.55	.0054	0.5000 − 0.4946	0.97
$ 200 up to $ 600	−2.55 up to −1.93	.0214	0.4946 − 0.4732	3.85
600 up to 1,000	−1.93 up to −1.31	.0683	0.4732 − 0.4049	12.29
1,000 up to 1,400	−1.31 up to −0.69	.1500	0.4049 − 0.2549	27.00
1,400 up to 1,800	−0.69 up to −0.07	.2270	0.2549 − 0.0279	40.86
1,800 up to 2,200	−0.07 up to 0.55	.2367	0.0279 + 0.2088	42.61
2,200 up to 2,600	0.55 up to 1.18	.1722	0.3810 − 0.2088	31.00
2,600 up to 3,000	1.18 up to 1.80	.0831	0.4641 − 0.3810	14.96
3,000 up to 3,400	1.80 up to 2.42	.0281	0.4922 − 0.4641	5.06
3,400 or more	2.42 or more	.0078	0.5000 − 0.4922	1.40
Total		1.0000		180.00

Once that is done, we can calculate the chi-square statistic.

Profit	f_o	f_e	$(f_o - f_e)$	$(f_o - f_e)^2$	$(f_o - f_e)^2/f_e$
Under $600	8	4.82	3.18	10.1124	2.098
$ 600 up to $1,000	11	12.29	−1.29	1.6641	.135
1,000 up to 1,400	23	27.00	−4.00	16.0000	.593
1,400 up to 1,800	38	40.86	−2.86	8.1796	.200
1,800 up to 2,200	45	42.61	2.39	5.7121	.134
2,200 up to 2,600	32	31.00	1.00	1.0000	.032
2,600 up to 3,000	19	14.96	4.04	16.3216	1.091
3,000 and over	4	6.46	−2.46	6.0516	.937
Total	180	180.00	0		5.220

Hypothesis Test that a Distribution is Normal Example Concluded

Step 1: State the null and alternate hypothesis

H_0: The population of profits follows the normal distribution.
H_1: The population of profits does not follow the normal distribution.

Step 2: Select the level of significance, we select .05

Step 3: Select the test statistic, we'll use chi-square, χ^2

Step 4: Formulate the decision rule, reject H_0 if $\chi^2 > 11.070$

Step 5: Make decision, $\chi^2 = 5.220$, we do not reject H_0

$$\chi^2 = \Sigma \frac{(f_o - fe)2}{f_e} = \frac{(8 - 4.82)^2}{4.82} + \cdots\cdots + \frac{(4 - 6.46)^2}{6.46} = 5.220$$

Step 6: Interpret, we conclude the evidence does not suggest the distribution of profits is other than normal.

Contingency Table Example

Example 1: Ford Motor Company operates the Dearborn plant with 3 shifts per day, 5 days a week. Vehicles are classified as to quality level (acceptable, unacceptable) and shift (day, afternoon, night). Is there a difference in the quality level on the three shifts?

Example 2: Rainbow Chemical, Inc. employs hourly and salaried employees. The vice president of human resources surveyed 380 employees about his/her satisfaction level with the current health care benefits program. The employees were then classified according to pay type, salary or hourly. Is it reasonable to conclude that pay type and level of satisfaction with the health care benefits are related?

Pay Type	Satisfied	Neutral	Dissatisfied	Total
Salary	30	17	8	55
Hourly	140	127	58	325
Total	170	144	66	380

Contingency Table

- We can use a contingency table to test whether two traits or characteristics are related
- The expected frequency will be determined as follows

$$\text{EXPECTED FREQUENCY} \qquad f_e = \frac{(\text{Row total})(\text{Column total})}{(\text{Grand total})} \qquad [15\text{–}5]$$

- The degrees of freedom = (Rows – 1)(Columns – 1)
- Example
- Ford Motor Company operates the Dearborn plant with 3 shifts per day, 5 days a week. Vehicles are classified as to quality level (acceptable, unacceptable) and shift (day, afternoon, night). Is there a difference in the quality level on the three shifts?

Contingency Table Example

Rainbow Chemical, Inc. employs hourly and salaried employees. The vice president of human resources surveyed 380 employees about his/her satisfaction level with the current health care benefits program. The employees were then classified according to pay type, salary or hourly. Is it reasonable to conclude that pay type and level of satisfaction with the health care benefits are related?

Pay Type	Satisfied	Neutral	Dissatisfied	Total
Salary	30	17	8	55
Hourly	140	127	58	325
Total	170	144	66	380

Step 1: State the null and alternate hypothesis

H_0: There is no relationship between level of satisfaction and pay type

H_1: There is a relationship between level of satisfaction and pay type

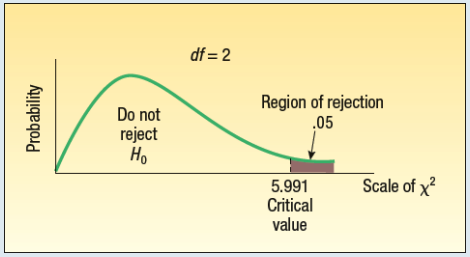
Step 2: Select the level of significance, we select .05

Step 3: Select the test statistic, we'll use chi-square, χ^2

Contingency Table Example Continued

Step 4: Formulate the decision rule, reject H_0 if chi-square > 5.991

Degrees of Freedom *df*	Right-Tail Area .10	.05	.02	.01
1	2.706	3.841	5.412	6.635
2	4.605	5.991	7.824	9.210
3	6.251	7.815	9.837	11.345
4	7.779	9.488	11.668	13.277
5	9.236	11.070	13.388	15.086



Step 5: Make decision, chi-square is 2.506, do not reject H_0

$$f_e = \frac{(\text{Row total})(\text{Column total})}{(\text{Grand total})} = \frac{(55)(170)}{380} = 24.61$$

	Satisfaction Level with Health Care						
	Satisfied		Neutral		Dissatisfied		
Pay Type	f_o	f_e	f_o	f_e	f_o	f_e	Total
Salary	30	24.61	17	20.84	8	9.55	55
Hourly	140	145.39	127	123.16	58	56.45	325
Total	170	170.00	144	144.00	66	66.00	380

$$\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e} = \frac{(30 - 24.61)^2}{24.61} + \frac{(17 - 20.84)^2}{20.84} + \cdots + \frac{(58 - 56.45)^2}{56.45}$$
$$= 1.181 + .708 + \cdots + .043 = 2.506$$

Step 6: Interpret, the sample data do not provide evidence that pay type and satisfaction level with health care benefits are related.

Elaboration

- **Elaboration** is a process designed to further explore a bivariate relationship; it involves the introduction of control variables.
- A **control variable** is an additional variable considered in a bivariate relationship. The variable is controlled for when we take into account its effect on the variables in the bivariate relationship.
- **Three Goals of Elaboration**
  1. Elaboration allows us to test for non-spuriousness.
  2. **spuriousness**, refers to a connection between two variables that appears causal but is not.
  3. Elaboration clarifies the causal sequence of bivariate relationships by introducing variables hypothesized to intervene between the IV and DV.
  4. Elaboration specifies the different conditions under which the original bivariate relationship might hold.

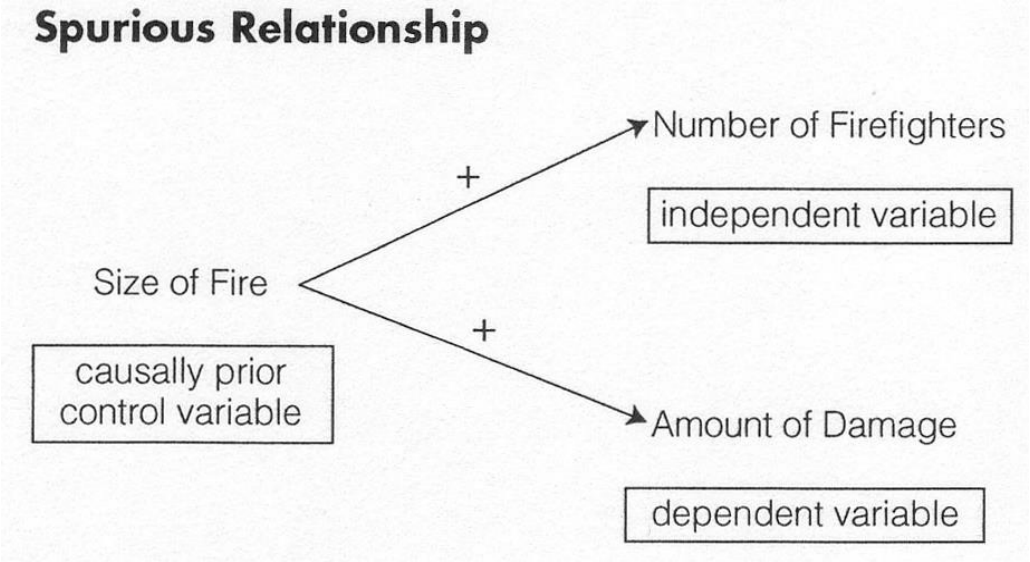
Testing for Nonspuriousness

- **Direct causal relationship:** a bivariate relationship that cannot be accounted for by other theoretically relevant variables.
- **Spurious relationship:** a relationship in which both the IV and DV are influenced by a causally prior control variable and there is no causal link between them. The relationship between the IV and DV is said to be "explained away" by the control variable.
- Example: The Bivariate Relationship Between Number of Firefighters and Property Damage

Number of Firefighters → Property Damage

(IV) (DV)

Testing for Nonspuriousness

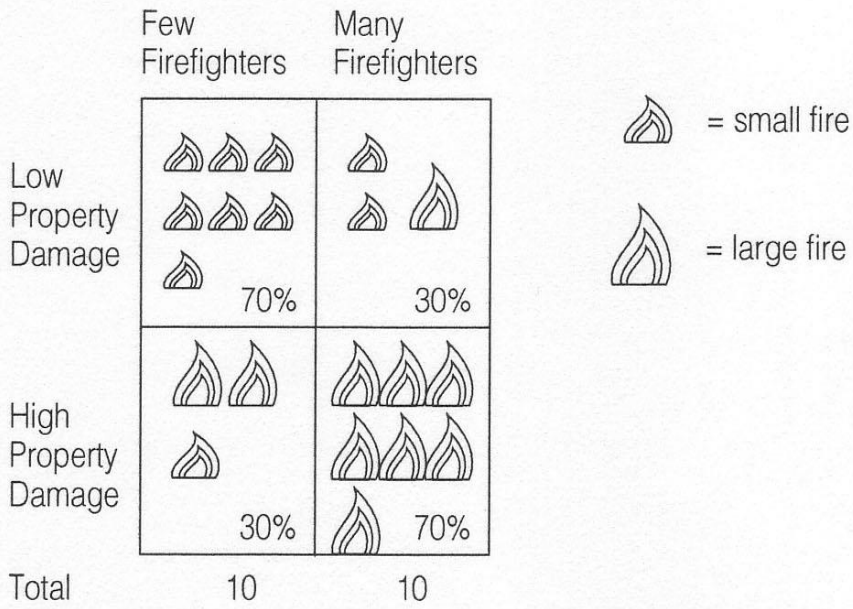


Process of Elaboration

- ***Partial tables:*** bivariate tables that display the relationship between the IV and DV while controlling for a third variable.
- ***Partial relationship:*** the relationship between the IV and DV shown in a partial table.
  1. Divide the observations into subgroups on the basis of the control variable. We have as many subgroups as there are categories in the control variable.
  2. Re-examine the relationship between the original two variables separately for the control variable subgroups.
  3. Compare the partial relationships with the original bivariate relationship for the total group.

1. A bivariate relationship between the number of firefighters and the extent of the property damage at 20 fire sites.

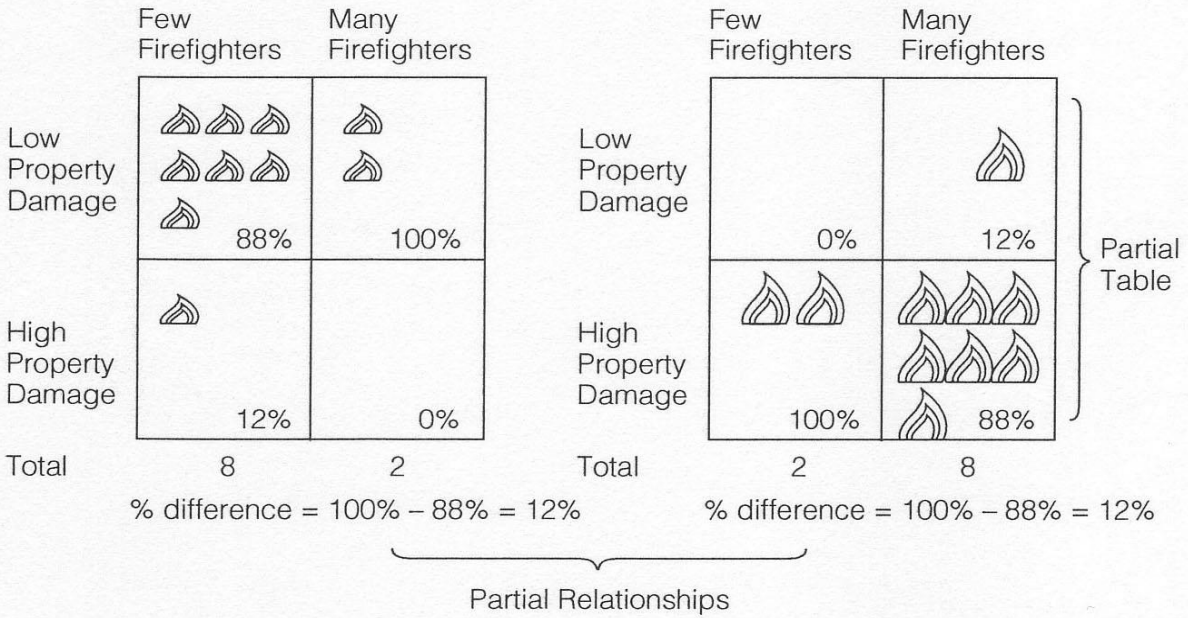
(a)



% difference = 70% – 30% = 40% (column percentages)

2. Control for size of fire: divide fire sites into small and large fires. In each group, recalculate the bivariate relationship between the number of firefighters and the extent of the property damage.

(b)



3. Compare the partial relationships with the original relationship: 40% compared with 12%.